Imperial College London

Topic 11

Operational Amplifier (op-amp)

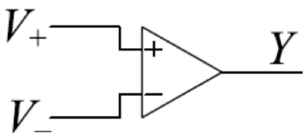
Professor Peter YK Cheung
Dyson School of Design Engineering

URL: www.ee.ic.ac.uk/pcheung/teaching/DE1_EE/
E-mail: p.cheung@imperial.ac.uk

Ideal Operational Amplifier

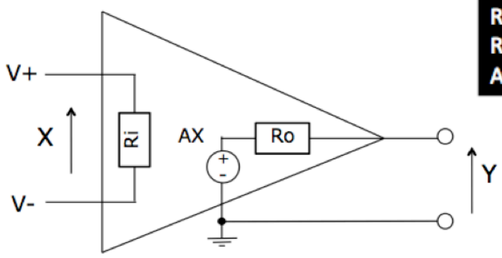
- An op-amp (operational amplifier) is a circuit with two inputs and one output.

  $Y = A(V+ - V-)$



- The equivalent circuit of an op-amp is shown here.
- For an ideal op-amp, Ri = ∞, Ro = 0, A = ∞

Ideal op--amp
Ri = ∞
Ro = 0
A = ∞



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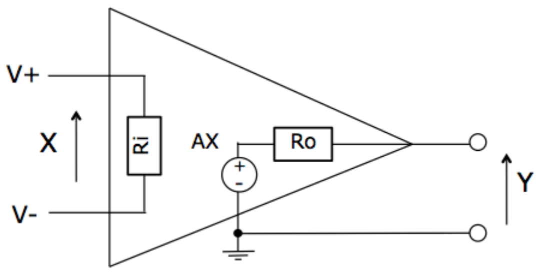
Operational amplifier is one of the most common electronic building blocks used by engineers. It has two input terminals: V^+ and V^-, and one output terminal Y. It amplifies X to give Y, i.e. Y = AX.

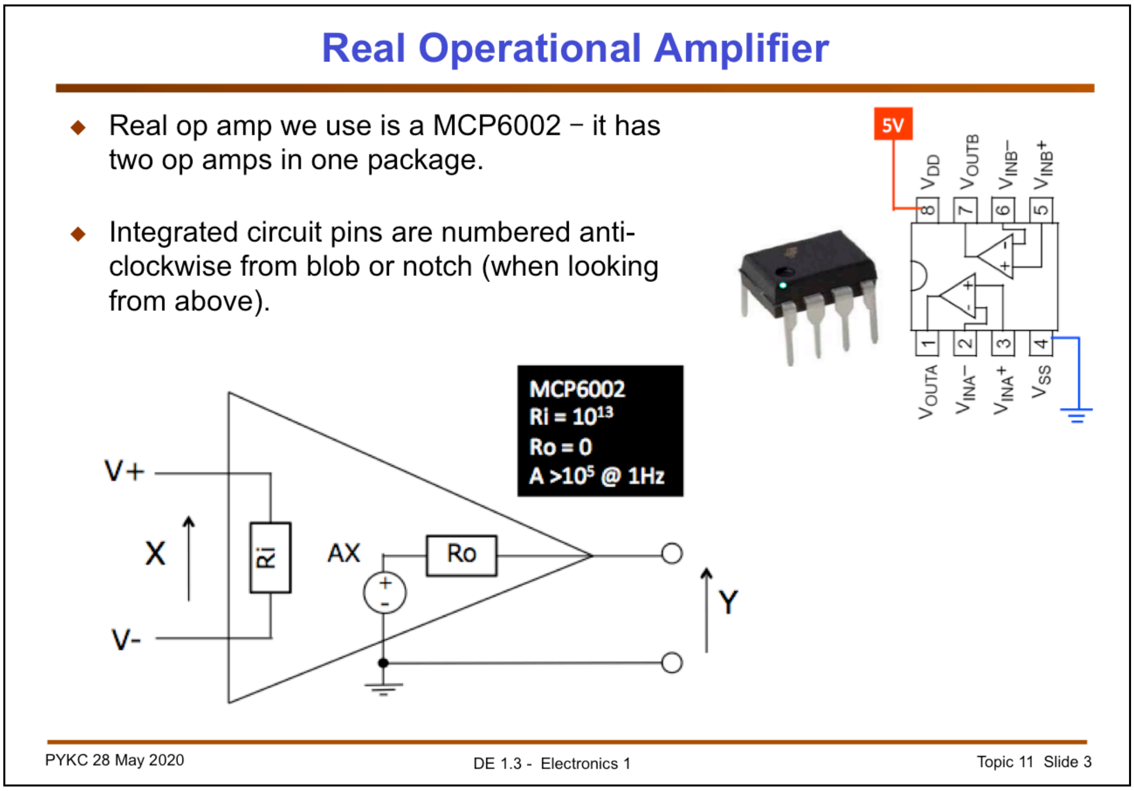
An ideal op-amp has infinite input impedance Ri, zero output impedance Ro and an infinite gain A. For an ideal op-amp, for all possible output Y, X is assumed to be zero because A is infinite.

We can model an op amp as shown here:

X is the voltage difference = $(V^+ - V^-)$.

Ideal op--amp
Ri = ∞
Ro = 0
A = ∞





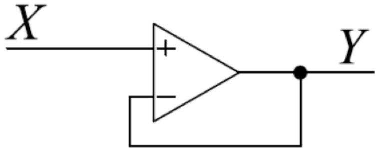
The output Y is AX, where A is the gain. Since A is very large, X is generally very small. For example, if the output voltage Y is +5V, A is 10^5, X is only 50μV. The scope you have been using at home cannot measure down to this voltage level.

An op-amp is actually very complex inside. However, as a user of op amp, and if you use it properly, you can simply ignore its internal complexity and treat it more or less like a perfect amplifier (i..e amplifying the difference voltage, and it is called a differential amplifier).

Op-amps belong to a type of electronic components known as integrated circuits (ICs). The packaging is as shown here where pin 1 is always on the left of the notch in the package and/or indicated with a dot.

Negative Feedback

- In a central heating system, if the temperature falls too low the thermostat turns on the heating, when it rises the thermostat turns it off again.
- ***Negative feedback*** is when the occurrence of an event causes something to happen that counteracts the original event.
- If op-amp output *Y falls* then $V-$ will fall by the same amount so $(V+ - V-)$ will increase.
- This causes Y to rise since



$$Y = A(V_+ - V_-).$$
$$Y = A(X - Y)$$
$$Y(1 + A) = AX \quad \Rightarrow \quad Y = \frac{1}{1+1/A}X \quad \rightarrow X$$ for large A

- If $Y = A(V_+ - V_-)$ then $V_+ - V_- = Y/A$ which, since $A \approx 10^5$, is normally *very very* small.

Golden Rule: **Negative feedback adjusts the output to make $V_+ \approx V_-$.**

Before we consider how to build an amplifier with an op-amp, let us consider the concept of **negative feedback**. Simply put, when an event causes something to change, the change itself will counteract the original event.

Shown here is the op-amp using negative feedback. The output Y is connect to the V_- input of the op amp. If output Y falls (the event), it will cause V_- to fall (the change). However, V_- falling will increase the difference voltage ($V_+ - V_-$). This causes Y to rise, thus counteracting the initial fall in Y.

The calculation shown here demonstrate that provided A is large, Y = X. Connecting Y to the V_- input ensures that this is always true with this circuit.

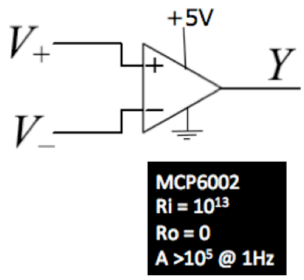
Note the golden rule: by feeding the output back to the **NEGATIVE** input of the op-amp which has very large gain, the circuit makes sure that ($V_+ - V_-$) approaches zero!

Analysing op-amp circuits

- Nodal analysis is simplified by making some assumptions.

Note: The op-amp often need two power supply connections. In our case, +5V and GND.

These are almost always omitted from the circuit diagram. The currents only sum to zero (KCL) if all five connections are included.



1. **Check for negative feedback:** to ensure that an increase in Y makes $(V_+ - V_-)$ decrease, Y must be connected (usually via other components) to $V-$.
2. **Assume** $V+ = V-$: Since $(V+ - V-) = YA$, this is the same as assuming that $A = \infty$. Requires negative feedback.
3. **Assume zero input current:** in most circuits, the current at the op-amp input terminals is much smaller than the other currents in the circuit, so we assume it is zero.
4. **Apply KCL at each op-amp input node separately** (input currents = 0).
5. **DO NOT apply KCL at output node** (output current is unknown).

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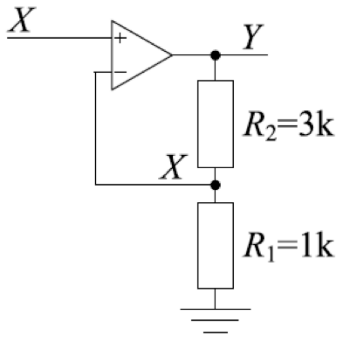
Op-amp requires power supply for it to work. For the sake of simplicity, we assume that the op-amp uses dual voltage supply with a +ve and a –ve voltage source as shown here. We will relax this assumption later.

To conduct analysis on op-amp circuits, we have to make some assumptions as shown here.

Non-inverting amplifier

- Circuit has input voltage X and output voltage Y .
- The circuit gain $\triangleq \frac{Y}{X}$.
- Applying steps 1 to 3:
  1. Negative feedback OK.
  2. $V_- = V_+ = X$
  3. Zero input current at V_- means $R2$ and $R1$ are in series (⇒ same current) and form a voltage divider. So $X = \frac{R_1}{R_1+R_2} Y$.

$$Y = \frac{R_1+R_2}{R_1} X = \left(1 + \frac{R_2}{R_1}\right) X = +4X$$



- *Non-inverting amplifier* because the gain Y/X is positive.
  - Consequence of X connecting to V_+ input.
  - Can have any gain ≥ 1 by choosing the ratio R_2/R_1.
- *Cause/effect reversal:* Potential divider causes
  - Feedback inverts this so that $Y = 4V_+$.

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Shown here is one of the most commonly used op-amp circuit to provide non-inverting amplification.

We check through the steps considered in the previous slide. This circuit has negative feedback. Input voltage X is connected to V_+. Therefore the voltage at V_- is also X.

Input current is zero. Therefore X is the voltage divider of Y as shown here.

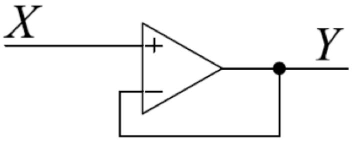
For such non-inverting op-amp circuit, Gain is always given by:

$$\text{Gain} = \frac{Y}{X} = (1 + \frac{R_2}{R_1})$$

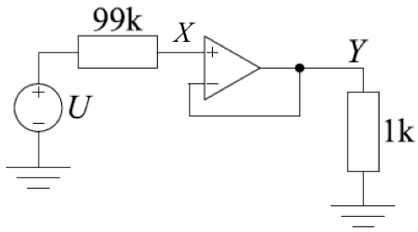
For the circuit shown here, the gain is x 4.

Voltage Follower

- A special case of the non-inverting amplifier with $R_1 = \infty$ and/or $R_2 = 0$.
- Gain is $1 + R_2/R_1 = 1$.
- Output Y "follows" input X.



- **Advantage**: Can supply a large current at Y while drawing almost no current from X. Useful if the source supplying X has a high resistance.
- **Without voltage follower:** $Y = 0.01U$.
- **With voltage follower:** $Y = U$.



- Although the *voltage gain* is only 1, the *power gain* is much larger.

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There is a special case for the non-inverting op-amp circuit. If you make R2 = 0, and remove R1, then the gain is 1. In fact even if R2 is larger than zero (say 1k), the ratio R2/R1 is still zero since R1 is infinite.

This circuit is known as voltage follower, or voltage buffer – output Y always follow input X.

This circuit may first appear pointless – Y is the same as X, why not just use X in the first place?

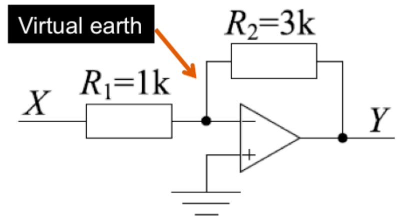
The reason why this circuit is useful is because it **ISOLATES** the output from the input by presenting a high impedance to the source (Ri is high) and low impedance to the load, hence behaving like an ideal amplifier, but with gain of 1.

Let us consider the example here. The source voltage U has a resistance of 99kΩ. If we connect this directly to a 1kΩ load, the output Y = 0.01U (voltage divider principle). The drop of voltage is due to the loading effect of the 1k resistor on the source.

When you put the voltage follow between U and the 1k load resistor, the source U sees the very high input impedance of the op-amp (>10MW), therefore the input X is effective U. The output resistance of the op-amp is low. The negative feedback also helps. If the loading effect of the 1k resistor causes Y to drop, this will cause V- input to drop, and raising Y, thus correcting the loading effect.

Inverting Amplifier

- Negative feedback OK.
- Since $V_+ = 0$, we must have $V_- = 0$.
- KCL at V_- node:



$$\frac{0-X}{R_1} + \frac{0-Y}{R_2} = 0 \quad \Rightarrow \quad Y = -\frac{R_2}{R_1}X = -3X.$$

- *Inverting Amplifier* because gain Y/X is negative. Consequence of X connecting to the V_- input (via R_1).
  - Can have any gain ≤ 0 by choosing the ratio R_2/R_1.
  - Negative feedback holds V_- very close to V_+.
  - If $V_+ = 0V$, then V_- is called a **virtual earth** or **virtual ground**.
- Nodal Analysis: Do KCL at V_+ and/or V_- to solve circuit. When analysing a circuit, you never do KCL at the output node of an op-amp because its output current is unknown. The only exception is if you have already solved the circuit and you want to find out what the op amp output current is (e.g. to check it is not too high).

This is the circuit for an inverting amplifier using an op-amp. Applying KCL at the V_- node gives the following equation:

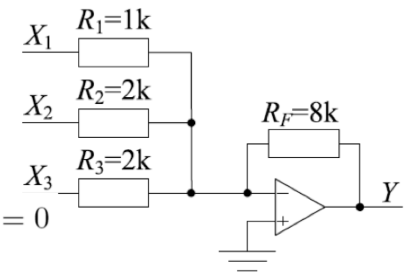
$$\text{Gain} = \frac{Y}{X} = -\left(\frac{R_2}{R_1}\right)$$

Since the V_- node is at the same voltage potential of the V_+ node, which is ground (or earthed), we call V_- node in this circuit the virtual earth or virtual ground.

Unlike the non-inverting amplifier case, which MUST have a gain ≥1, inverting amplifier like this can have any gain, larger or smaller than 1. However, the gain is ALWAYS negative.

Inverting Summing Amplifier

- We can connect several input signals to the inverting amplifier.
- As before, $V_- = 0$ is a virtual earth due to negative feedback and $V_+ = 0$.



- KCL at V_- node: $\frac{0-X_1}{R_1} + \frac{0-X_2}{R_2} + \frac{0-X_3}{R_3} + \frac{0-Y}{R_F} = 0$

$$\Rightarrow \quad Y = -\left(\frac{R_F}{R_1}X_1 + \frac{R_F}{R_2}X_2 + \frac{R_F}{R_3}X_3\right)$$

$$\Rightarrow \quad Y = -(8X_1 + 4X_2 + 4X_3).$$

- Y is a weighted sum of the input voltages with the weight of X_i equal to

$$\frac{R_F}{R_i} = G_i R_F.$$

- **Input Isolation**: The current through R_1 equals $\frac{X_1 - 0}{R_1}$ which is not affected by X_2 or X_3. Because V_- is held at a fixed voltage, the inputs are isolated from each other.

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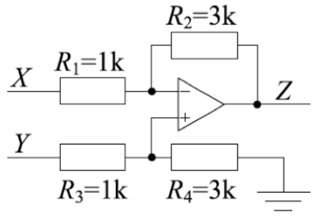
With inverting amplifier, since the V- node is virtually zero, you can connect multiple sources to this node via a resistor and produce a summing amplifier.

Apply KCL at V- node and you see the summing effect immediately. Each voltage is weighted by the ratio of the feedback resistor R_F and the feeding resistor R_i (i.e. the weighting is - R_F/R_i).

The fact that the V- node is held at a fixed voltage (in this case 0v, but it could have been a different voltage, as we will see later), the effect of input sources X1, X2 etc is isolated from each other.

Differential Amplifier



- A 2-input circuit combining inverting and non-inverting amplifiers.
- Linearity ⇒ $Z = aX + bY$.
- Use superposition to find a and b.
- Find a: Set $Y = 0$. KCL at V_+ node ⇒ $V_+ = 0$. We now have an inverting amplifier, so $Z = -R2/R1$ $Z = -3X$ ⇒ $a = -3$.
- Find b: Set $X = 0$. We can redraw circuit to make it look more familiar: a potential divider followed by a non-inverting amplifier.
- R_3 and R_4 are a potential divider (since current into V_+ equals zero), so

  $V_+ = \frac{R_4}{R_3+R_4}Y = \frac{3}{4}Y$.
- The non-inverting amplifier has a gain of $\frac{R_1+R_2}{R_1} = 4$.
- The combined gain is $b = \frac{R_4}{R_3+R_4} \times \frac{R_1+R_2}{R_1} = \frac{3}{4} \times 4 = +3$.
- Combining the two gives $Z = 3\,(Y - X)$. The output of a *differential amplifier* is proportional to the diffference between its two inputs.

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We can combine the structure of the inverting AND non-inverting amplifier together to form this differential amplifier. The best way to understand this is to apply the principle of superposition.

Consider the effect of X on output Z with Y set to zero. The output due to X alone is:

$$Z_X = -\frac{R_2}{R_1}X$$

Consider the effect of Y on output Z with X set to zero. The output due to Y alone is more complicated. Firstly, Y is reduced by the voltage divider before reaching the V+ input.

$$V_+ = \frac{R_4}{R_3 + R_4}Y$$

This is amplified according to the non-inverting amplifier gain:

$$Z_Y = (1 + \frac{R_2}{R_1}) \times (\frac{R_4}{R_3 + R_4})Y$$

Now assume that $R_2 = R_4 = 3k$, and $R_1 = R_3 = 1k$.

$$Z_Y = (1 + \frac{R_2}{R_1}) \times (\frac{R_2}{R_1 + R_2})Y = \frac{R_2}{R_1}Y$$

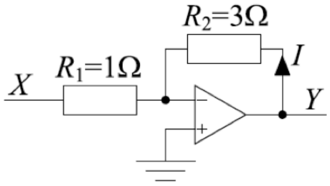
Therefore

$$Z = Z_X + Z_Y = (\frac{R_2}{R_1}) \times (Y - X) = 3 \times (Y - X)$$

Choosing Resistor Values

- The behaviour of an op-amp circuit depends on the ratio of resistor values:
- $Gain = -R_2/R_1$. How do you choose between 3Ω/1Ω, 3kΩ/1kΩ , 3MΩ/1MΩ and 3GΩ/1GΩ?

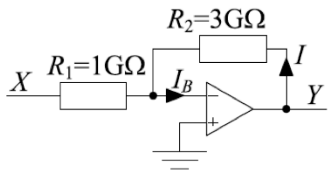
- Small resistors cause large currents.
  - If $X = \pm 1V$, then $Y = \mp 3V$,
    and $I = \frac{Y-0}{R_2} = \mp 1\,\text{A}.$
  - However typical op-amps can only supply $\pm 5mA$, so the circuit **will not work**.



- Large resistors increase sensitivity to interference and to op-amp input currents.
- If the bias current into V_- is $I_B = 1\ nA$, then KCL at V_- gives

$$\frac{0-Y}{R_2} + \frac{0-X}{R_1} + I_B = 0 \Rightarrow Y = -\frac{R_2}{R_1}X + I_B R_2 = -3X + 3$$

instead of $Y = -3X$.



- Within wide limits, the absolute resistor values have little effect. However you should avoid extremes.

So far, we assume that the op-amp behaves like an ideal amplifier with infinite gain A, infinite input resistance Ri, and zero output impedance R_O.

These assumptions only hold if the resistors we use to construct the circuit are sensible. So how do we choose these resistor values?

If we use too low a resistor value, the output current required is too much – no good.

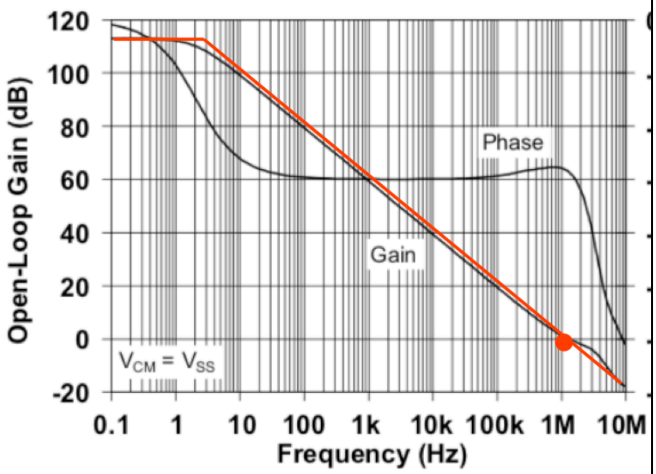
If we use too high a resistor value, the input current into the op-amp is no longer negligible. Then the gain equation we derived using KCL at the input node is no longer valid.

In general, we use resistor values in 1kΩ to 100kΩ region.

In the next slide, we will consider the assumption of infinite gain A.

Bandwidth of real op-amp

- The gain of an op-amp is very high at low frequency, but it decreases rapidly as the signal frequency increases as shown in the Gain vs Frequency plot for our op-amp.
- The gain at 1Hz is more than 10^5.
- The corner frequency is around 10 Hz.
- The gain then drops off like a RC characteristic, at around -20dB/decade (or x 0.1 / decade).
- Op-amps are characterised by the frequency at which the gain becomes unity. This is known as the **unity gain bandwidth**.
- In the case of MCP6002, this is approximately 1MHz.



For a practical op-amp, the gain is only very high at low frequency. Shown here is the Gain vs Frequency characteristic of our MCP6002 op-amp.

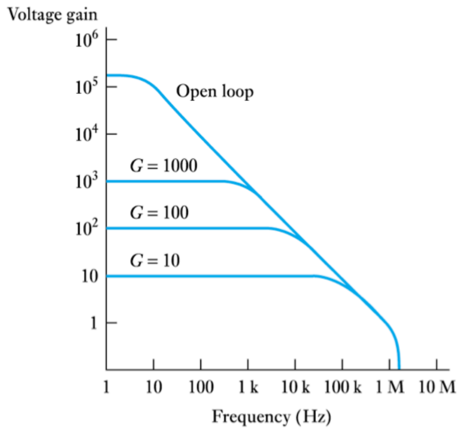
The gain is over 100,000 at frequency below 10Hz. However, the characteristic is similar to the of an RC low pass filter. The corner frequency is only 10Hz. It then falls off at -20dB (or x 0.1) / decade. The gain at 1MHz becomes around 1 (i.e. it stop behaving as an amplifier).

Benefits of negative feedback

- Using negative feedback in our op-amp circuit help to improve bandwidth.
- As the gain of the amplifier is reduced, the bandwidth is increased due to negative feedback.
- Without proving it on this course, for op-amp with negative feedback,

GAIN x BANDWITH = CONSTANT

- This is known as the **gain-bandwidth product** of the op-amp.
- For MCP6002 opamp we use in Lab3, the gain-bandwidth product is around 1MHz.
- Since this product is constant, if the gain is reduced, the bandwidth is increased. This is shown in the graph here.



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The detail analysis of the impact of this falling gain on bandwidth when we use negative feedback is beyond the scope of this module. However, it is worth stating the following observation:

Due to negative feedback, the produce of the overall circuit gain and the effective bandwidth (i.e maximum frequency for this to behave like an amplifier) is a constant. This product:

Gain x Bandwidth = gain-bandwidth product = constant.

For MCP6002, the product is around 1MHz, or 10^6.

The result of this is shown in the plot here.

Summary

- Ideal properties:
  - Zero input current
  - Infinite gain
  - Do not use KCL at output (except to determine output current).
- Negative Feedback circuits:
  - Assume $V_+ = V_-$ and zero input current
  - Standard amplifier circuits:
    - Non-inverting $gain = 1 + R_2/R_1$
    - Inverting $gain = -R_2/R_1$
    - Summing amplifier
    - Differential Amplifier
- Positive feedback circuits:
  - $VOUT = \pm V_{max}$ (no good for an amplifier)
  - Schmitt Trigger: switches when $V_+ = V_-$.
- Choosing resistors: not too low or too high.